FINITE ELEMENT PREDICTION OF FRACTURE TOUGHNESS OF COMPOSITE LAMINATES THROUGH J-INTEGRAL APPROACH

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C E R T I F I C A T E

This is to certify that the thesis entitled "FINITE ELEMENT PREDICTION OF FRACTURE TOUGHNESS OF COMPOSITE LAMINATES THROUGH J-INTEGRAL APPROACH" by B.K.MISHRA is a record of work carried out under our supervision and has not been submitted elsewhere for a degree.

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NOMENCLATURE

A	Area
a	Crack length; half crack length in centrally located crack and double edge notched plates.
a_{ij}	Terms of compliance matrix
$[B]$	Strain-displacement matrix
C	Slope of load-displacement curve
DEN	Double edge notched
$[D]_i$	Stress-strain matrix of i^{th} ply
$[D]_c$	Equivalent stress-strain matrix for composite laminates.
$\{d\}^{(ne)}$	Displacement matrix for element nodes
E_L	Longitudinal elastic modulus
E_T	Transverse elastic modulus
I	Potential energy function
J	J-integral
J_c	Critical value of J-integral
K	Stress intensity factor
k	Ratio of J-integral and square of applied displacement
$[k]^{(e)}$	Stiffness matrix for the finite element
N_i	Interpolation functions
n_j	Unit normal vector to J-integral path
$\{P\}^{(e)}$	Force matrix for the finite element
SEN	Single edge notched

s	Distance along J-integral path
T_i	Traction vector
t_i	Thickness of i^{th} ply
t_c	Thickness of composite laminate
U	Strain energy function for the laminate
U_i	Strain energy function for i^{th} ply
u_i	Displacement vector
u, v	Displacement in x and y direction
W	Width of the specimen
Γ	J-integral path
ε_{ij}	Strain tensor
σ_{ij}	Stress tensor
ν_{LT}	Major Poisson's ratio
δ	Applied displacement
η_i	Load relaxation factor at i^{th} ply failure
G_{LT}	In plane shear modulus
$\lfloor \quad \rfloor$	A row matrix
$\{ \quad \}$	A column matrix

ABSTRACT

The J-integral for quasi-isotropic $(0/\pm 45/90)_{2s}$ and crossplied $(0/90)_{4s}$ laminates has been evaluated using finite element technique. Constant strain traingular finite elements have been used. The program gives displacement at all the nodes and stresses and strains in all the elements. Stresses are obtained for all the plies and J-integral can be obtained for all the laminae as well as for the laminate.

Three crack configurations namely, single edge notched (SEN), double edge notched (DEN)and centrally located crack specimens have been analysed.

For the formulation of the problem, an equivalent stress-strain matrix for the laminate has been defined such as to model the laminate by a homogenous and orthotropic material. A stepwise method has been used to predict laminate failure. To account for effect of stacking sequence, models with no load relaxation and complete load relaxation in failed plies have been investigated.
The results obtained by present study compare quite well with the experimental investigation on SEN specimens.

CHAPTER I

INTRODUCTION

1.1 INTRODUCTION TO FRACTURE OF COMPOSITE MATERIALS

Composite materials are engineering combination of two or more materials to achieve certain physical properties not realizable by constituent materials individually. Composite materials have high strength and stiffness coupled with low density. In addition, they have good vibration, fatigue and corrosion resistance, low heat conductivity, good electrical insulation and favourable cost effectiveness. Manufacture of composites requires less labour and generates less waste. It has the additional advantage of easy processibility.

Since the composite materials are being increasingly used in many sophisticated applications, material selection criteria and design procedures are required. Global properties like deformation and modulus are well understood. Strength criteria, although not well established, are generally agreed upon. So, research emphasis is towards understanding of fatigue and fracture response of composites.

Fracture mechanics has now been widely accepted as a useful discipline, which deals with inherent macroscopic cracklike defects in materials. It deals with the study of fracture of materials by crack initiation and propagation under various service conditions. It aims at providing guidelines for tolerable size of cracklike defects and

inspection requirements for avoiding catastrophic failures. For metals and ceramics, developments in fracture mechanics has led to establishing methods of predicting fracture toughness. The fibre reinforced composites also seek reliable means of characterizing fracture behaviour as a part of establishing the fundamental properties.

Due to heterogeneity of the composite materials and complexity of interaction between constituents , fracture is accompanied by several processes. These processes include fibre breakage, fibre extraction, debonding between fibres and matrix, delamination, fracture of matrix etc. In some cases composites fracture during stretching along the fibres by breakdown of integrity. Even when steady growth of crack is observed, they differ substantially from the cracks in traditional materials. The problem of characterizing fracture properties of composite materials is challenging as the crack propagation in them may not be self-similar. Due to these complexities parallel or combined experimental and theoretical analyses is required.

1.2 LITERATURE REVIEW

In fracture mechanics two basic approaches, namely, the energy and stress approaches have emerged. The energy approach is due to Griffith [1]. He derived a fracture criterion by taking into account total change in energy of a cracked body. The stress approach is due to Irwin [2], who relied on stress analysis of a cracked body.

Definitive features of fracture in homogeneous isotropic solids were identified by Griffith [1] and extended by Orowan [3] and Irwin [2] for engineering materials. Irwin carried out stress analysis by assuming the material to be linear elastic. This approach is known as linear elastic fracture mechanics (LEFM).

Rice [4,5] used a different energy approach applicable to linear as well as nonlinear elastic materials and under conditions of large scale yielding also. He used a two dimensional energy line integral around crack tip, now better known as J-integral. This integral characterizes the crack tip. Basis of J-integral is provided by the works of Hutchinson [6] and Rice and Rosengren [7] . McClintock [8] has demonstrated that crack tip stress and strain fields can be described in terms of J-integral. The use of J-integral as an elastic-plastic fracture criterion has been discussed by Broberg [9]. Bagley and Landes [10-12] through extensive experiments have shown applicability of J-integral as fracture criterion for metals.

Several researchers applied linear elastic fracture mechanics to composite material and found that fracture toughness was not independent of initial crack length and they used a two parameter LEFM model. By a consideration of hole size effect. Nuismer and Whitney [13,14] proposed alternative approaches for fracture of laminated composites.

These two approaches are point stress criterion and average stress criterion. Giare [15] and Agarwal and Giare [16] found notched strength of short fibre composites to be in agreement with Whitney-Nuismer criteria.

The J-integral approach was attempted on glass fabric reinforced polyester resins by Smith, Green and Bowyer [17]. It has been used to short glass fibre composites by Agarwal, Patro and Kumar [18,19]. Srinivasan [20] has used J-integral for quasi-isotropic and crossplied laminates.

Babu [21] in a theoretical study has evaluated J-integral for short fibre and unidirectional composites using finite element method. He used a 2-dimensional displacement finite element analysis using constant strain traingular elements. J-integral for randomly oriented short fibre composites and unidirectional composites was evaluated for plane strain and plane stress cases.

He conducted the analysis for three different crack configuration, namely, single edge notched, double edge notched and centrally located crack. His check for path independency of J-integral was satisfactory. J_c for short fibre composites are in good agreement with experimental works of Patro [22].

1.3 SCOPE OF PRESENT WORK

The present studies are an attempt to predict the fracture behaviour of composite laminates using J-integral as a fracture criterion. Symmetric laminates have been analysed using equivalent stiffness matrix for the laminates.

J-integral has been calculated for individual laminae through lamina stiffness matrix. Attempt has been made to predict first ply failure and analysis after first ply failure has been carried out by modified stiffness matrix.

The analysis has been carried out for quasi-isotropic $[0/\pm 45/90]_{2s}$ and crossplied $[0/90]_{4s}$ laminates for three different crack configurations, namely, single edge notched, double edge notched and centre located cracked.

Formulation of the problem is given in chapter 2. Finite element solution technique adopted for the present study is also discussed in chapter 2. Chapter 3 deals with the numerical results obtained. The various models adopted to predict fracture behaviour of laminates are also discussed in this chapter, in the light of experimental findings. Conclusions and some suggestion for future work are included in chapter 4. The computer program used for the present study has been given as an appendix.

CHAPTER II

J-INTEGRAL AS FRACTURE CRITERION FOR COMPOSITE MATERIALS

2.1 INTRODUCTION

Most of the research work on fracture of composite materials, so far uses linear elastic fracture mechanics (LEFM) concepts and requires accurate stress analysis at the crack tip. Due to stress singularity at crack tip, this is difficult even for homogenous isotropic materials. In heterogenous composites, the situation becomes more difficult due to complex crack tip geometry.

The J-integral is a parameter which can be evaluated, due to its path independency, without focussing attention at the crack tip, but can still characterize the crack tip. J-integral does not require a prescribed crack extension trajectory and is hence more adaptable to characterize cracks in composite material.

The J-integral, as proposed by Rice [4,5] is a two dimensional energy line integral defined as

$$J = \int_{\Gamma} \left(U dy - T_i \frac{\partial u_i}{\partial x} ds \right) \qquad (2.1)$$

where Γ is any contour traversing in counter clockwise sense and enclosing the crack tip as shown in Fig. 2.1, U the strain energy density is defined as

$$U = U(\varepsilon_{mn}) = \int_{0}^{\varepsilon_{mn}} \sigma_{ij} \, d\varepsilon_{ij}$$

Where σ_{ij} and ε_{ij} are stress and strain tensors

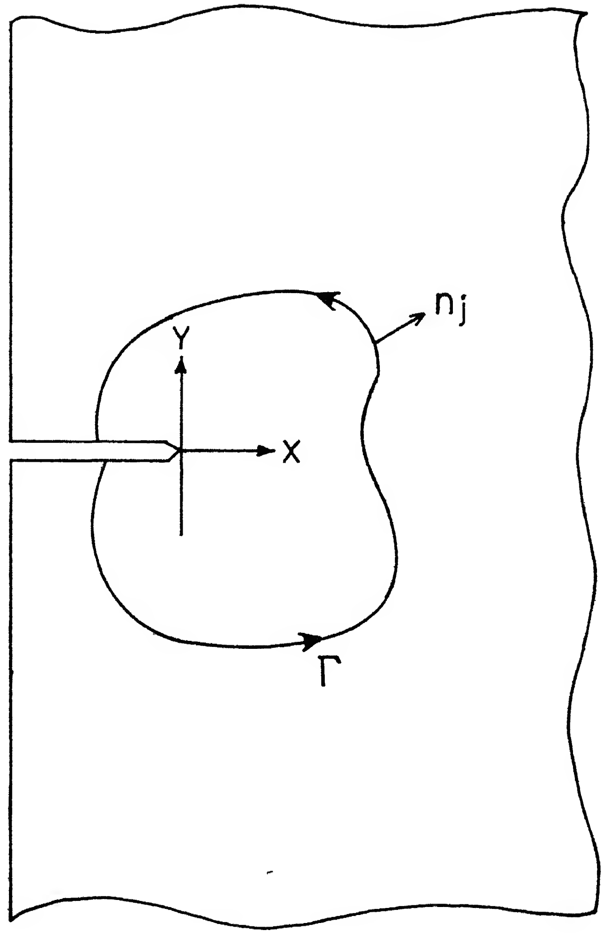

FIG. 2.1 A CRACK TIP MODEL

respectively. T_i is the traction vector and is given by

$$T_i = \sigma_{ij}\, n_j$$

where n_j is the normal vector to Γ. u_i is the displacement vector and s the arc length along Γ.

2.2 FORMULATION OF PROBLEM

For evaluation of J-integral, stresses, strains and derivatives of displacement are to be found out. To achieve this end total potential energy stored in the laminate has been minimized by Raleigh -Ritz method [23]. The finite element program, then solves the problem with prescribed boundary conditions to yield displacement at all the nodes, their derivatives, strains and stresses.

2.2.1 Expression for Potential Energy

For symmetric laminates subjected to inplane loads only, displacement and strain fields are same throughout the thickness of the laminate, and thus donot vary from lamina to lamina [24].

Due to different fibre direction, stress-strain matrix vary from lamina to lamina and hence the stresses in them are not same even with same strain field.

Now if the stress-strain relationship of i^{th} lamina in a laminate be $[D]_i$ and its thickness be t_i, strain energy stored in it is given by

$$U_i = \frac{1}{2}\int_{v_i} \lfloor \varepsilon \rfloor_i \{\sigma\}_i \, dv = \frac{1}{2}\int_{v_i} \lfloor \varepsilon \rfloor_i \, [D]_i \{\varepsilon\}_i \, dv \quad (2.2)$$

where $\lfloor \varepsilon \rfloor$ is the row matrix $\lfloor \varepsilon_1 \quad \varepsilon_2 \quad \gamma_{12} \rfloor$ and $\{\varepsilon\}$ is the column matrix. So the total strain energy stored in the laminate will be

$$U = \sum_i U_i = \int_V \lfloor \varepsilon \rfloor \left(\sum_i [D]_i \, dv_i \right) \{\varepsilon\} \tag{2.3}$$

Now if the laminate is a flat plate

$$dv_i = dA \, t_i \tag{2.4}$$

and eqn. 2.3 can be written as

$$U = \int_A \lfloor \varepsilon \rfloor \left(\sum_i [D]_i \, t_i \right) \{\varepsilon\} \, dA \tag{2.5}$$

Now an equivalent stress strain matrix for the laminate is defined such that

$$[D]_c \, t_c = \sum_i [D]_i \, t_i \tag{2.6}$$

where $[D]_c$ is the equivalent stress-strain matrix and t_c is the total laminate thickness.

Equation 2.5 can be rewritten as

$$U = \int_A \lfloor \varepsilon \rfloor [D]_c \{\varepsilon\} \, t_c \, dA \tag{2.7}$$

Now in the absence of body forces expression for the potential energy of the laminate can be written as

$$I = \int_A \lfloor \varepsilon \rfloor [D]_c \{\varepsilon\} \, t_c \, dA - \int_s T_j \, u_j \, ds \tag{2.8}$$

2.2.2 Boundary Condition

Three different crack configurations namely single edge notched, double edge notched, and centrally located crack plate have been analysed. These plates have been subjected to mode I loading condition as shown in Fig. 2.2. Boundary condition in all of them are

Along AB $\quad V = 0, \quad \tau_{xy} = 0$

Along BC and DA $\quad \sigma_x = 0\ ; \quad \tau_{xy} = 0$ (Traction free)

Along CD $\quad V = \delta$ (applied displacement)

and $\tau_{xy} = 0$

crack surfaces are stress free.

2.3 FINITE ELEMENT SOLUTION TECHNIQUE

Expression for the potential energy of the laminate, Eq. 2.8, contains only first derivative of displacements. So compatibility of u,v and completeness of $u,\ v,\ \frac{\partial u}{\partial x},\ \frac{\partial v}{\partial x},\ \frac{\partial u}{\partial y},\ \frac{\partial v}{\partial y}$ are required [23].

Traingular elements with three nodes with following interpolation function satisfy the above requirements. Thus

$$\{u\}^e = a + bx + cy = \lfloor N_1 \quad N_2 \quad N_3 \rfloor \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\{v\}^e = a' + b'x + c'y = \lfloor N_1 \quad N_2 \quad N_3 \rfloor \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \end{Bmatrix} \qquad (2.9)$$

where $\lfloor N_1 \quad N_2 \quad N_3 \rfloor$ are interpolation functions.

Substituting equation 2.9 in 2.8 and applying the Raleigh-Ritz method [23], one gets

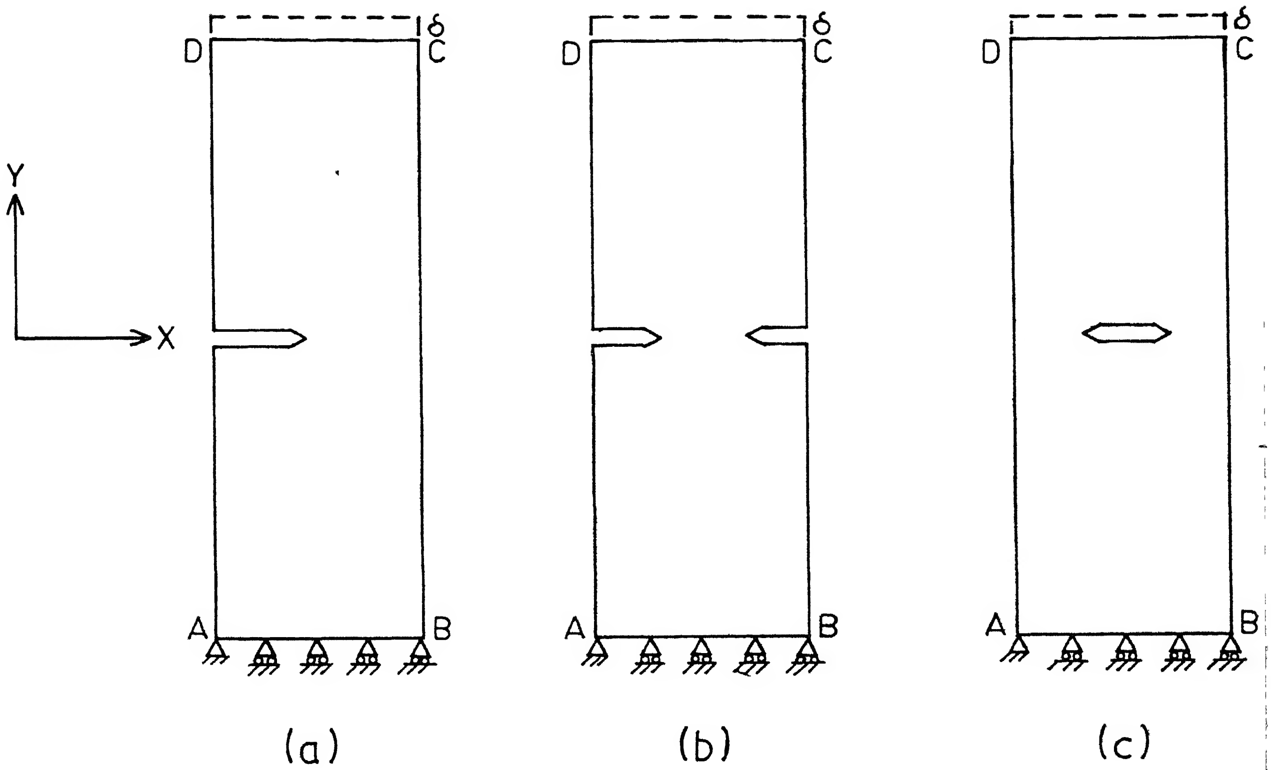

FIG. 2.2 THREE CRACK CONFIGURATIONS WITH BOUNDARY CONDITIONS.

$$[k]^{e} \; \{d\}^{(ne)} = \{P\}^{(ne)} \tag{2.10}$$

where $[K]^{e} = \iint_{A(e)} [B]^{T} \; [D]_{c} \; [B] \; t_{c} \; dA$ = stiffness matrix,

$$[B]_{i} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad \text{and} \quad \{d\}^{(ne)} = \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

i is the node number.

So for traingular elements

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} \end{bmatrix} \tag{2.11}$$

and
$$\{d\}^{ne} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \tag{2.12}$$

$$\left\{ P \right\}^{ne} = \int_s p \left\{ N_B \right\} ds = \text{Load matrix} \tag{2.13}$$

N_B = interpolation function along the boundaries of the finite element.

Now the specimen to be analysed is divided into a finite number of traingular elements. A moderately fine element grid is maintained near crack tip. The size of elements is increased as it is moved away from crack tip because of less variation in stress. Element stiffness and load matrices are obtained and assembled. After applying appropriate boundary condition, displacement, derivatives of displacement and strains are obtained. After this using the lamina properties stresses in different laminae and consequently J is determined for all laminae seperately. In view of the path independency, contour is chosen along the boundary of the specimen as it is convenient.

2.4 A CORRECTION FACTOR

For homogenous and orthotropic material, relation between J and stress intensity factor K is given by [25]

$$J = \frac{K^2}{E'} \tag{2.14}$$

where $$\frac{1}{E'} = \left(\frac{a_{11} \ a_{22}}{2} \right)^{1/2} \left[\left(\frac{a_{22}}{a_{11}} \right)^{1/2} + \frac{a_{66} + 2a_{12}}{2a_{11}} \right]^{1/2} \tag{2.15}$$

a_{ij} are terms of the compliance matrix of the material.

Now if the elastic modulus in loading direction be E_1 then

$$J = \left(\frac{E_1}{E'}\right) \frac{K^2}{E_1} \qquad (2.16)$$

Now **from** the finite element formulation only $\frac{K^2}{E_1}$ factor is evaluated, so a correction factor $\frac{E_1}{E'}$ is introduced to get J value for the laminate.

CHAPTER III

RESULTS AND DISCUSSION

This chapter deals with the numerical results obtained for quasi-isotropic and crossplied laminates and their comparison with experimentally obtained values. An attempt has been made to predict laminate failure by using properties of constituent laminae. A stepwise method has been used to predict laminate failure.

3.1 PROCEDURE FOR LAMINATE ANALYSIS

For a given specimen, J-integral was evaluated at different displacements for the laminate as well as for the laminae, constituting the laminate. At small displacement none of the plies fails. As the load increases, plies start failing. A ply was assumed to fail when the applied displacement exceeded a critical displacement for the ply. The final laminate fracture is assumed at the critical displacement for the laminate obtained experimentally. The basic lamina properties and the critical displacements for the purpose of present investigation were obtained experimentally and are given in Table 3.1.

After a ply fails, its contribution to the overall laminate behaviour changes. This change is incorporated in the analysis through modification of its stiffness matrix. There are different approaches which can be used to modify the stiffness matrix [24]. One approach suggests that the stiffness matrix of a failed lamina should be made zero. This would mean that the lamina would carry no additional load in

any direction. This will be a conservative approach since, for example, when a ply fails by transverse tensile failure, it may not carry any additional load in transverse direction but can certainly carry additional load in the fiber direction A more practical approach under such circumstances would be to assume that while the transverse tensile modulus and shear modulus of a failed ply are zero but the longitudinal modulus remains unaffected. The latter approach has been used in the present analysis.

TABLE 3.1
LAMINA ELASTIC CONSTANTS

Longitudinal modulus, E_L	=	31.9 GPa
Transverse modulus, E_T	=	9.7 GPa
Major poisson's ratio, ν_{LT}	=	0.25
Shear modulus, G_{LT}	=	3.2 GPa

Laminates	Critical displacement (mm)
0° unidirectional	0.88
90° unidirectional	0.08
45° unidirectional	0.10
Quasi-isotropic, $(0/\pm 45/90)_{2s}$	0.75*
Crossplied, $(0/90)_{4s}$	0.375*

* From reference 20

The J-integral evaluated through a stepwise procedure may be represented by a piecewise continuous curve as shown in Fig. 3.1, where each discontinuity point represents a ply failure. In the segment between two discontinuities the J-integral, $(J)_i$, can be written as follows

$$(J)_i = (J)_{i-1} + (\Delta J)_i \qquad (3.1)$$

Where $(J)_{i-1}$ is the value of J at the $(i-1)^{th}$ discontinuity and $(\Delta J)_i$ is the increamental J in i^{th} segment. When all the plies are intact, the J-integral is evaluated using the original stiffness matrix for the laminate. The increamental J is calculated using modified stiffness matrix.

Another point which must be considered at this point is what happens to the load carried by the ply when it fails That is, when a ply fails what is the extent to which the load carried by it drops. In a real situation, it may be between the two extreme situations, namely, one in which the load in failed ply drops to zero (complete load relaxation) and two, the load doesnot drop at all (no load relaxation). It can be easily shown that the later condition would show greater increament of J compared to the former condition for the same displacement increament. In the present investigations, results have been obtained for both the conditions.

3.2 J-INTEGRAL FOR LAMINATES

J-integral was evaluated as a function of applied displacement (δ) for quasi-isotropic and crossplied laminates. The ratio of crack length to specimen width was

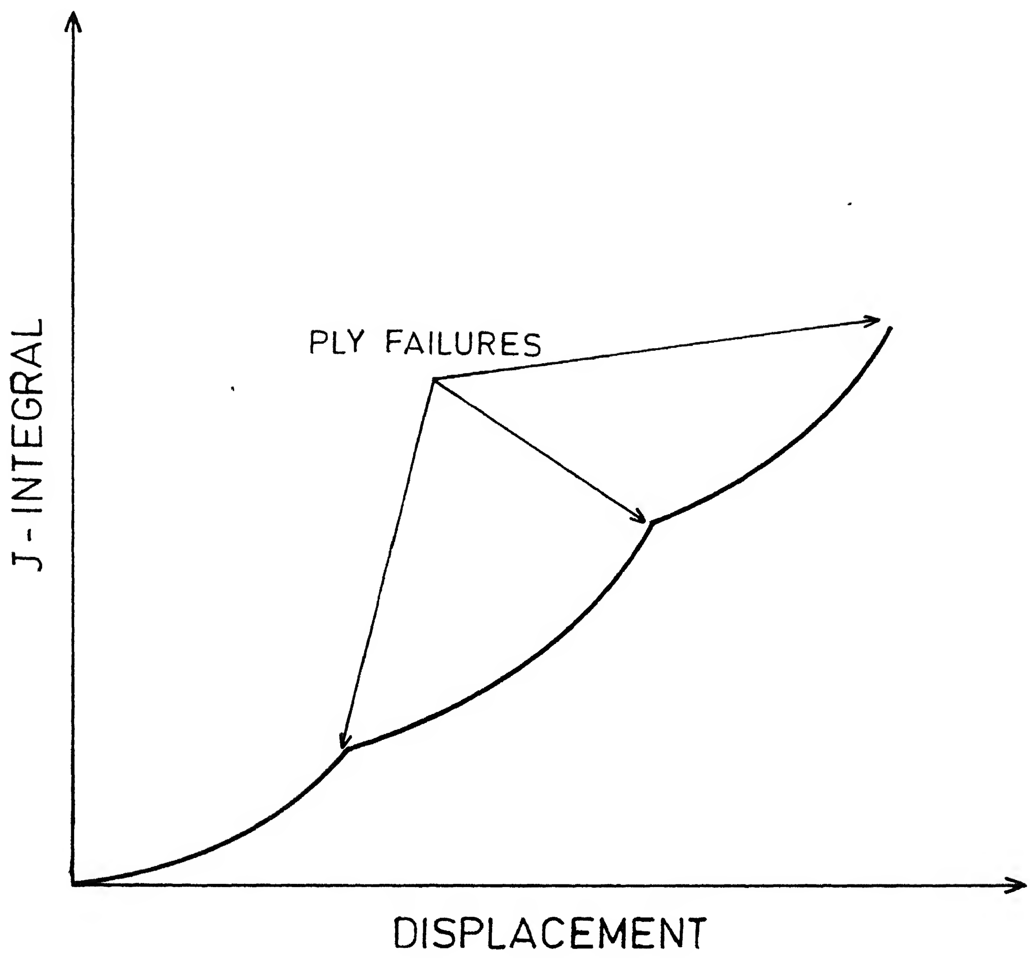

FIG. 3.1 J vs δ PLOT FOR A LAMINATE

varried from 0.1 to 0.7. The following crack configurations were used:

i) Single edge notched (SEN) specimens

ii) Double edge notched (DEN) specimens

iii) Centrally located notch specimen

J vs δ curves for SEN specimens are shown in Figs. 3.2 and 3.3 for quasi-isotropic and crossplied laminates respectively. J is plotted against δ^2 in Figs. 3.4 and 3.5. Since J vs δ^2 plots are straight lines passing through the origin, it is clear that J-integral is proportional to the square of applied displacement for all these cases. In fact, this behaviour is expected. In the present analysis, it has been assumed that the laminate stress-strain (or load-displacement) behaviour is linear. The load-displacement curves for a laminate with different crack lengths may be represented as shown in Fig. 3.6. For the linear relationship, the potential energy stored is given by

$$U = \frac{1}{2} c \, \delta^2 \tag{3.2}$$

where 'c' is the slope of P-δ cruve. It should be noted that c is a function of crack length and type of crack, but is independent of applied displacement 'δ'. Therefore J-integral at applied displacement 'δ' will be

$$J = -\frac{\partial U}{\partial a} = \left(-\frac{1}{2} \frac{\partial c}{\partial a} \right) \delta^2 = k \, \delta^2 \tag{3.3}$$

where $k = -\frac{1}{2} \frac{\partial c}{\partial a}$ is again a function of crack configuration only.

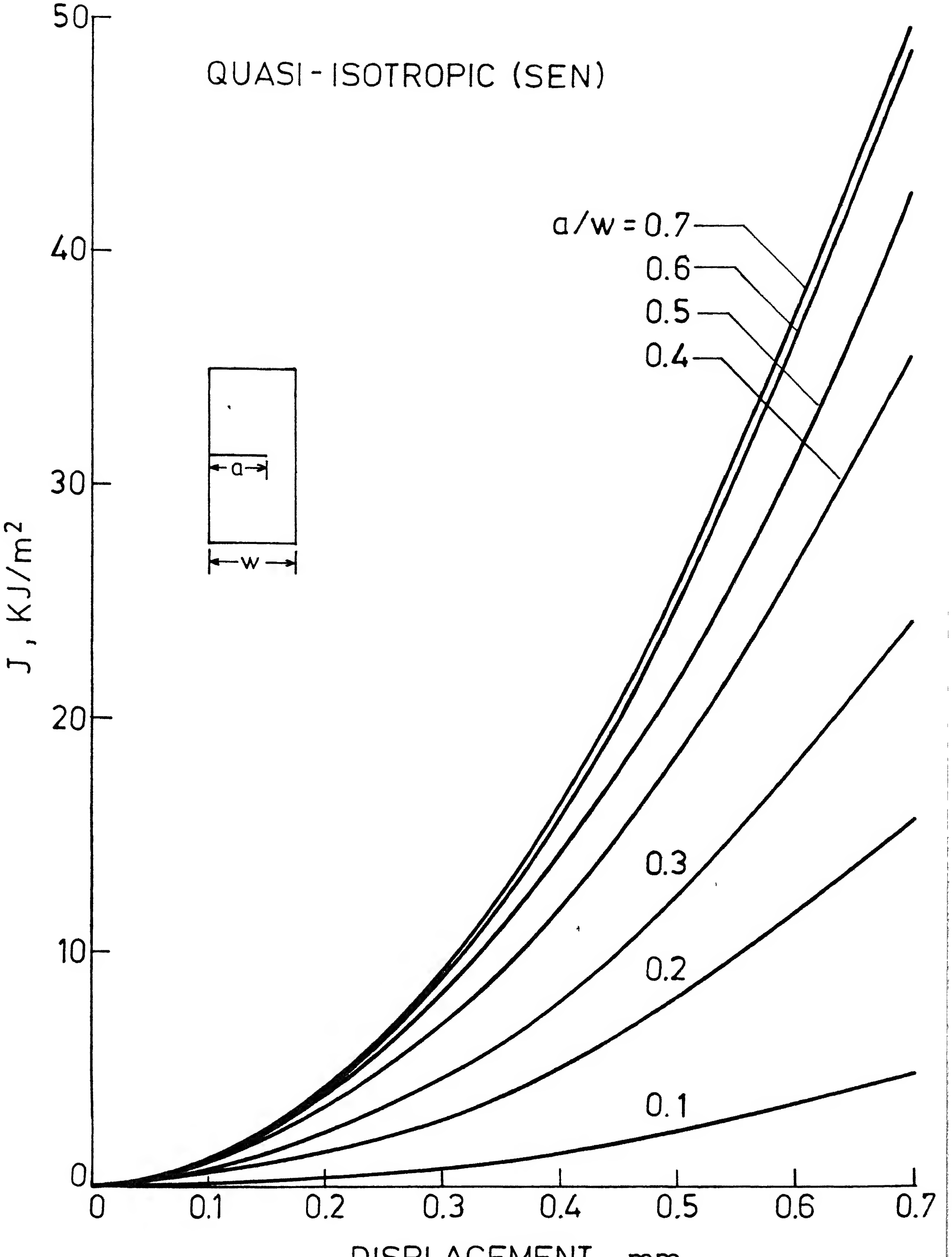

FIG. 3.2 J vs δ PLOT FOR DIFFERENT (a/w)

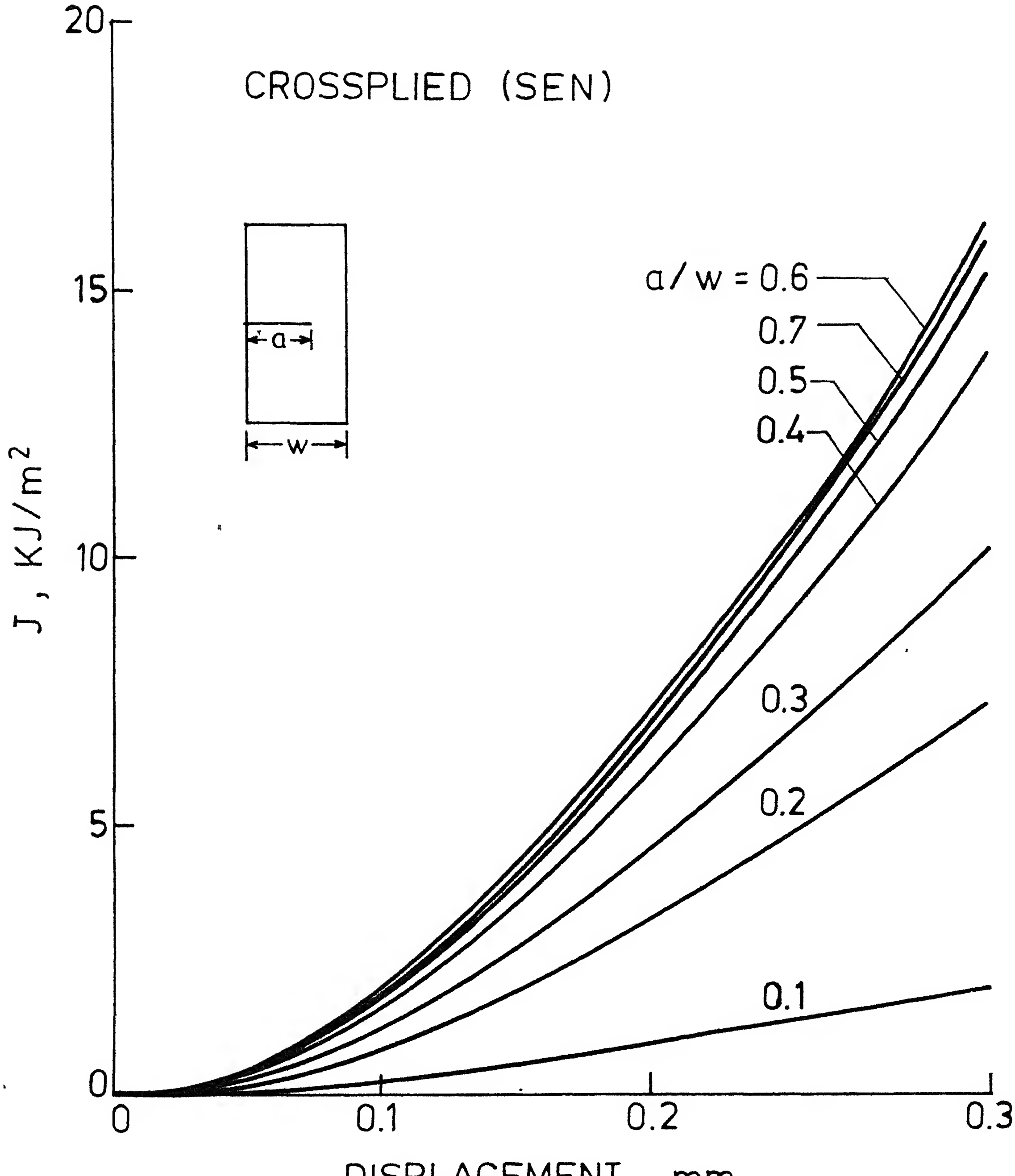

FIG. 3.3 J vs δ PLOT FOR DIFFERENT (a/w)

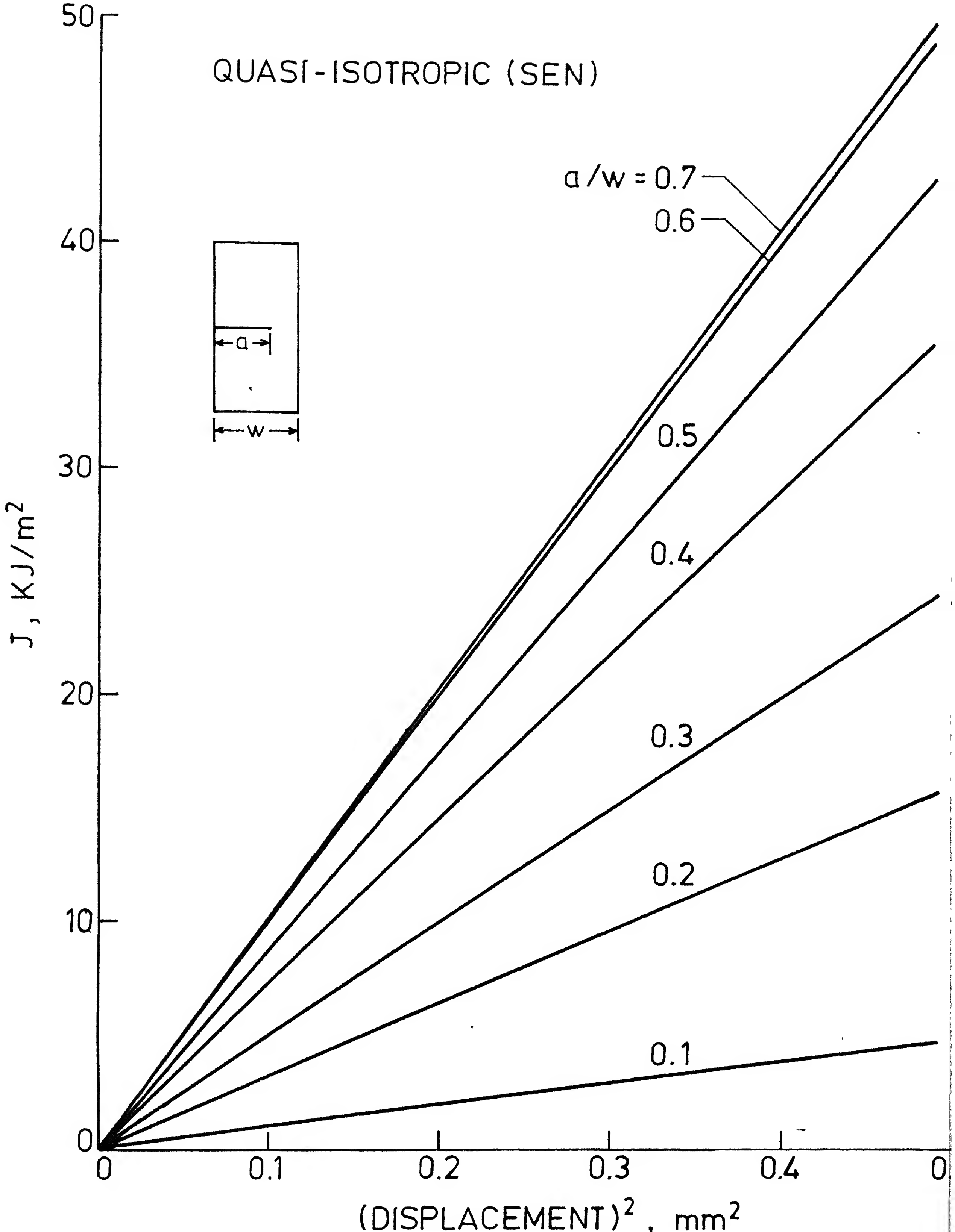

FIG. 3.4 J vs δ^2 PLOT

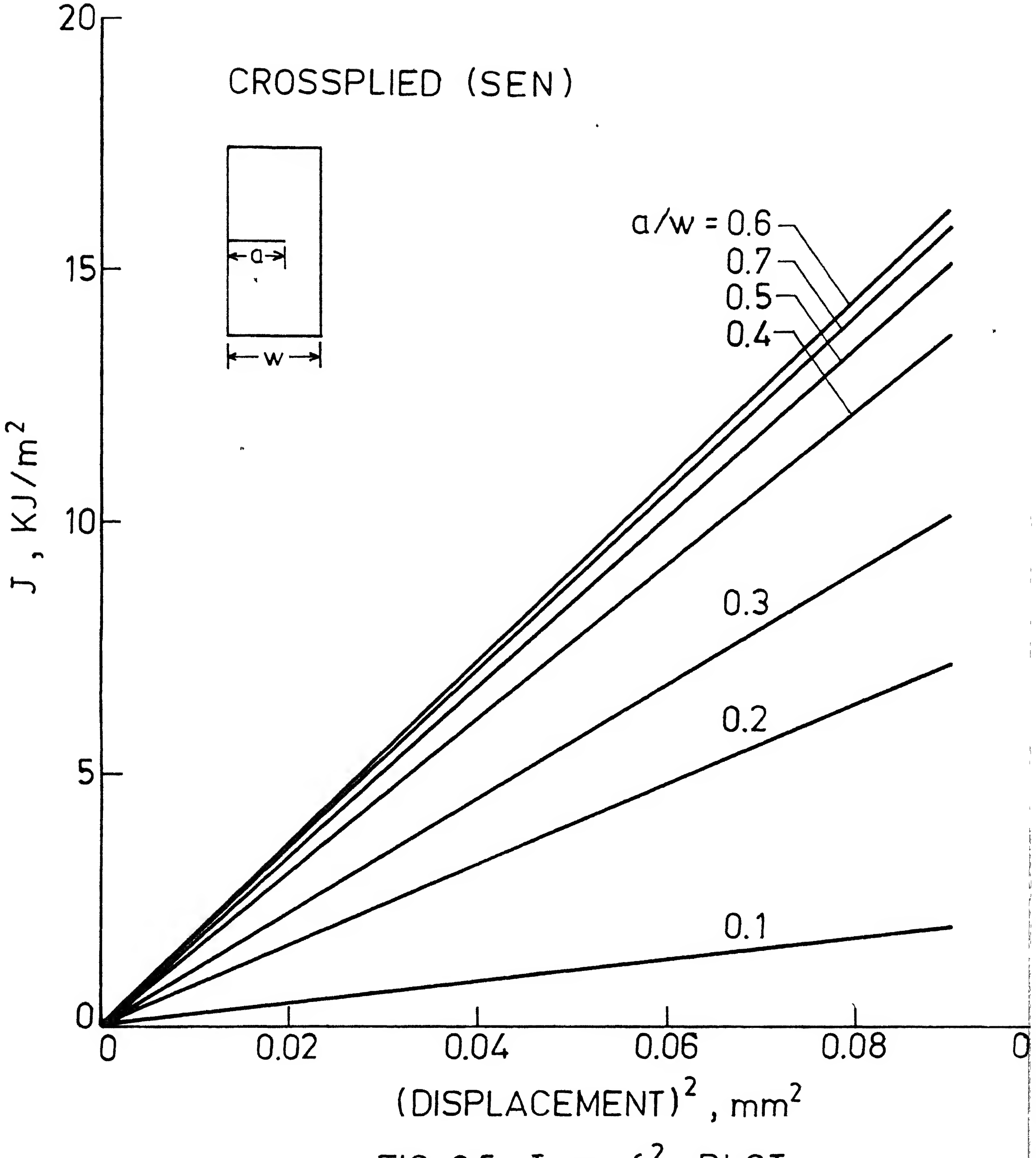

FIG. 3.5 J vs δ^2 PLOT

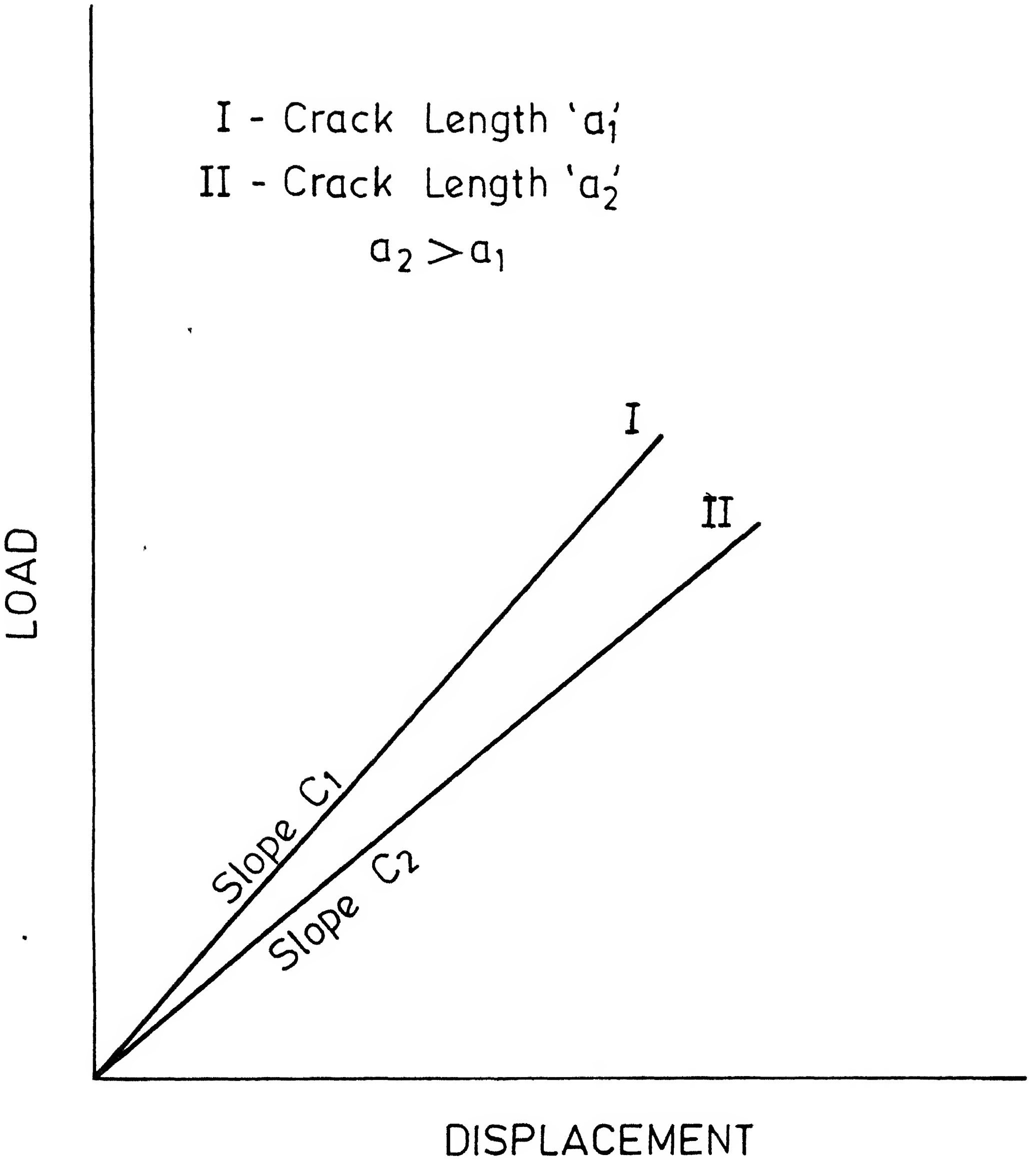

FIG. 3.6 LOAD DISPLACEMENT CURVES FOR DIFFERENT CRACK LENGTHS.

In view of the above discussion, the J-δ relationship can be studied, without any loss of information, through the parameter (J/δ^2) representing slope of J vs δ^2 lines in Figs. 3.4 and 3.5.

For a particular laminate with given crack configuration, the family of J-δ curves can be reduced to a single curve of (J/δ^2) plotted against crack length to specimen width ratio.

For the purpose of analysis, J/δ^2 for the laminate was obtained with its primary stiffness matrix. After a ply failure, (J/δ^2) values were again obtained with the modified stiffness matrix. This process was continued till the laminate failed.

Values of (J/δ^2) were obtained for quasi-isotropic and crossplied laminates having three different crack configurations, namely (i) SEN, (ii) DEN and (iii) centrally located crack. J/δ^2 values are given in Tables 3.2 to 3.7 and shown graphically in Figs. 3.7 to 3.12. The values have been shown for the laminate when all the plies are intact and also for the laminates after initial failure, i.e. when plies have failed in ascending order of their critical displacement. These curves are used to predict the overall laminate behaviour. It may be noticed (Figs. 3.2 and 3.3) that for SEN specimens J vs. δ curves for a/w ratios of 0.5, 0.6 and 0.7 are quite close to each other for quasi-isotropic as well as crossplied laminates. This is in good agreement with experimental observations that J-integral curve is independent of crack length when a/w >0.5 [18,20,22].

TABLE -3.2

Single Edge Notched

Quasi-isotropic Laminate

$\frac{a}{W}$	J/δ^2 in $kJ/m^2\text{-}mm^2$		
	ALL PLIES INTACT	90 PLIES FAILED	90 and $\pm 45^{\circ}$ PLIES FAILED
0.1	9.78	8.48	6.10
0.2	32.15	27.82	19.65
0.3	49.88	43.27	29.55
0.4	73.25	63.62	46.22
0.5	87.55	76.22	56.32
0.6	99.98	87.20	65.42
0.7	102.0	89.02	67.48

TABLE 3.3

Single Edge Notched

Crossplied Laminate

$\frac{a}{W}$	J/δ^2 in $kJ/m^2\text{-}mm^2$	
	ALL PLIES INTACT	90 PLIES FAILED
0.1	21.95	18.2
0.2	81.25	71.5
0.3	113.35	94.35
0.4	154.60	123.30
0.5	170.3	131.6
0.6	181.25	136.15
0.7	178.55	133.2

TABLE -3.4

Double Edge Notched

Quasi-isotropic Laminate

$\frac{2a}{W}$	J/δ^2 in kJ/m^2-mm^2		
	ALL PLIES INTACT	90° PLIES FAILED	90° and $\pm 45^\circ$ PLIES FAILED
0.1	6.83	5.92	4.28
0.2	14.27	12.28	8.67
0.3	20.88	17.95	12.77
0.4	27.18	23.41	16.68
0.5	33.09	28.42	20.27
0.6	39.35	34.06	24.28
0.7	46.71	40.52	29.00

TABLE 3.5

Double Edge Notched

Crossplied Laminate

$\frac{2a}{W}$	J/δ^2 in kJ/m^2 - mm^2	
	ALL PLIES INTACT	90° PLIES FAILED
0.1	15.35	12.51
0.2	36.27	32.24
0.3	51.14	44.70
0.4	66.74	58.61
0.5	79.53	68.91
0.6	94.27	80.96
0.7	108.25	91.04

TABLE -3.6

Centrally Located Crack

Quasi-isotropic Laminate

$\frac{2a}{W}$	J/δ^2 in $kJ/m^2\text{-}mm^2$		
	ALL PLIES INTACT	90^o PLIES FAILED	90^o and $\pm 45^o$ PLIES FAILED
0.1	5.90	5.12	3.66
0.2	12.31	10.66	7.47
0.3	18.51	16.02	11.27
0.4	25.51	22.08	15.56
0.5	32.98	28.54	20.25
0.6	42.59	36.82	26.34
0.7	54.21	46.88	33.91

TABLE -3.7

Centrally Located Crack

Crossplied Laminate

$\frac{2a}{W}$	J/δ^2 in $kJ/m^2\text{-}mm^2$	
	ALL PLIES INTACT	90^o PLIES FAILED
0.1	14.05	12.08
0.2	33.06	30.28
0.3	48.01	42.94
0.4	65.29	58.02
0.5	80.48	69.91
0.6	99.22	83.97
0.7	117.88	96.34

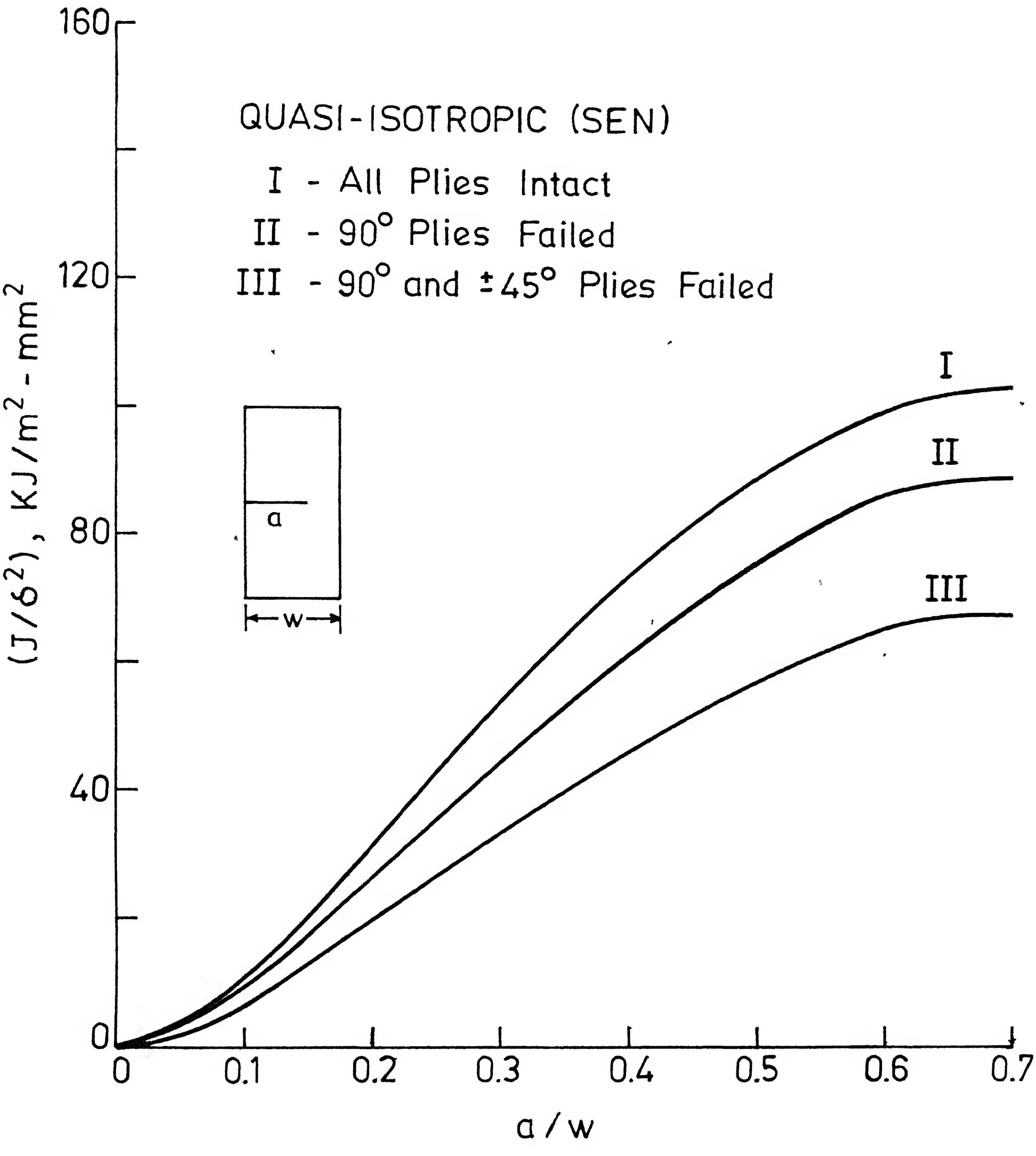

FIG. 3.7 J/δ^2 FOR QUASI-ISOTROPIC LAMINATE.

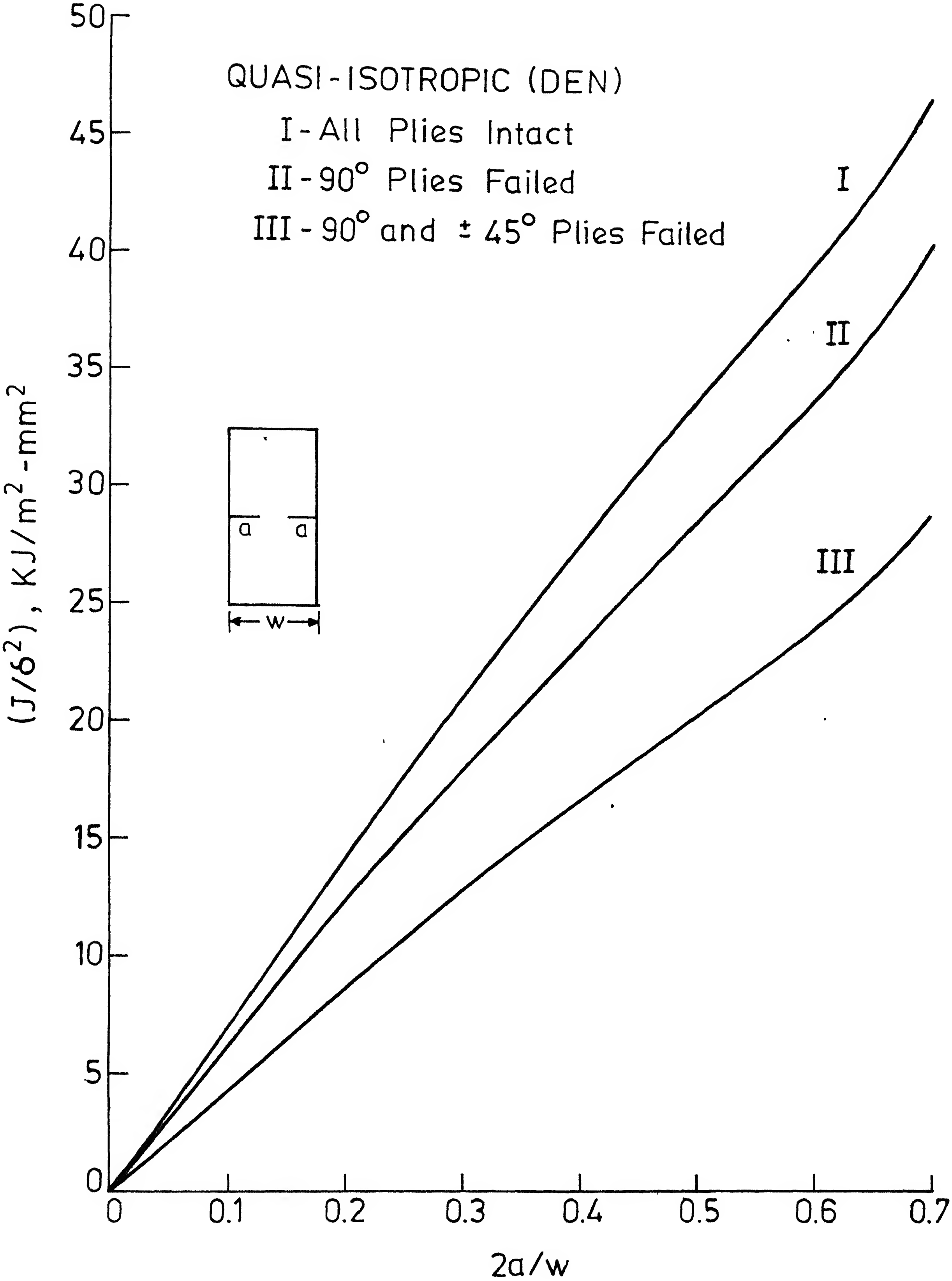

FIG. 3.8 J/δ^2 FOR QUASI-ISOTROPIC LAMINATE.

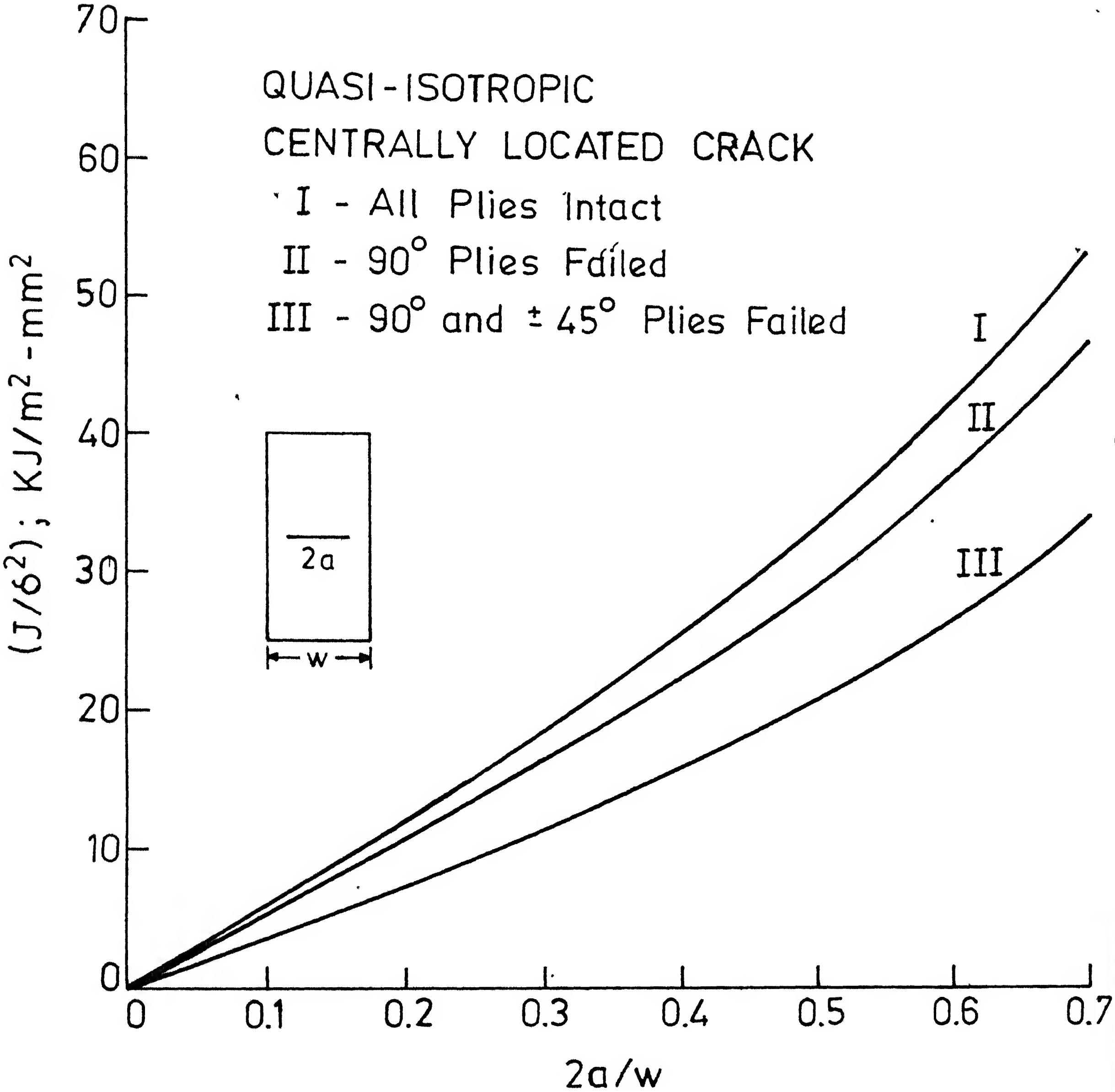

FIG. 3.9 J/σ^2 FOR QUASI-ISOTROPIC LAMINATE

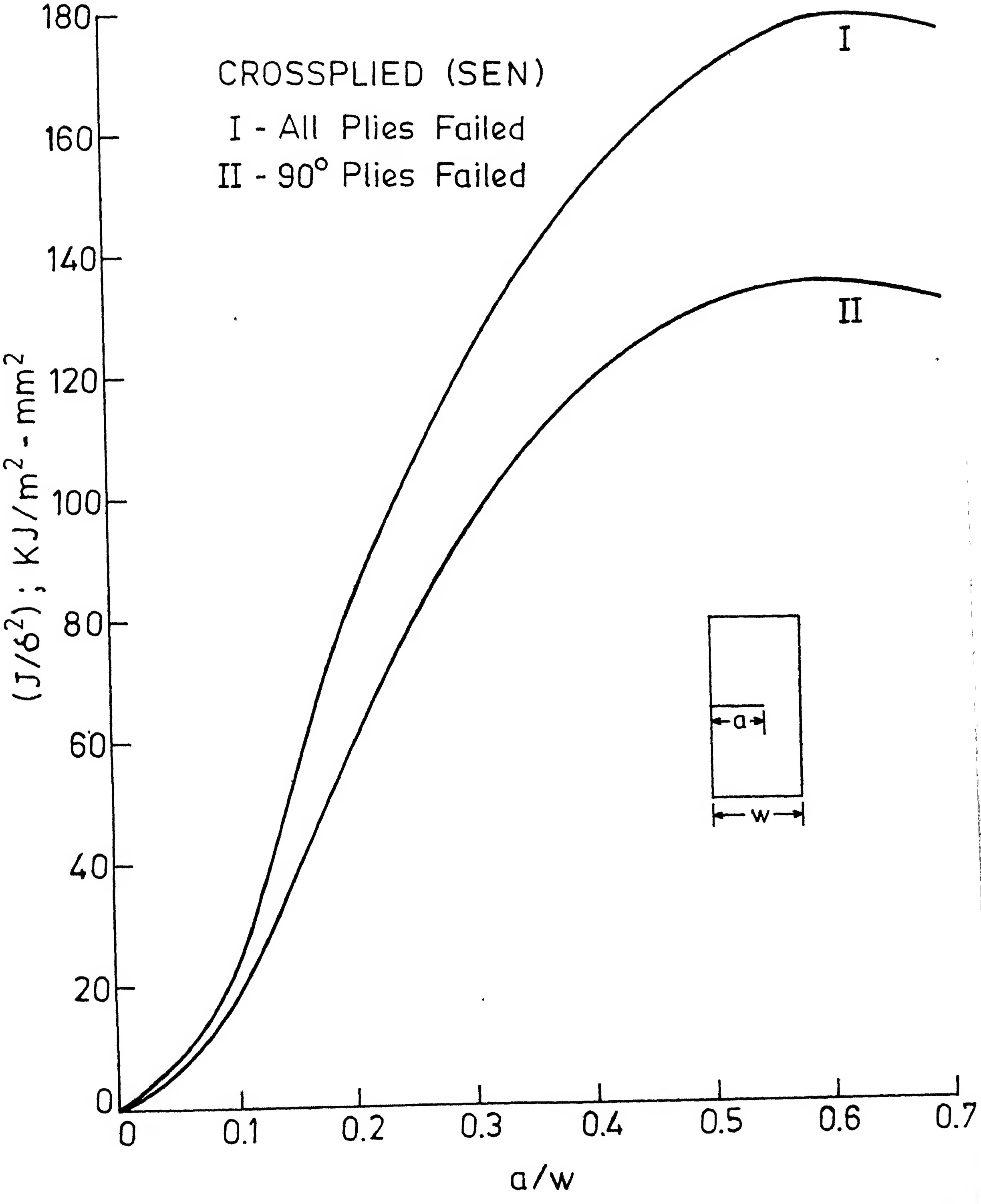

FIG. 3.10 J/σ^2 FOR CROSSPLIED LAMINATE.

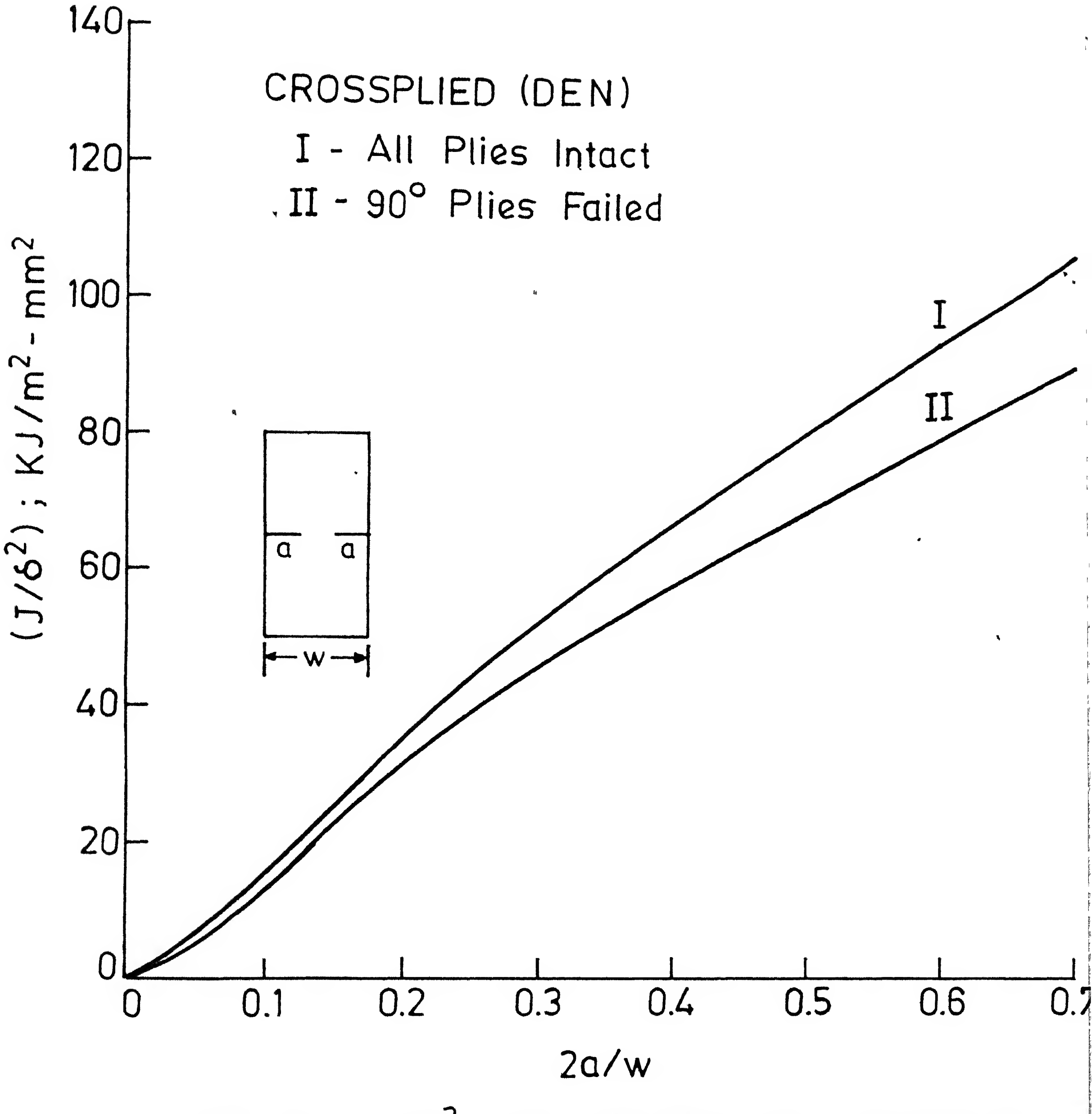

FIG. 3.11 J/δ^2 FOR CROSSPLIED LAMINATE.

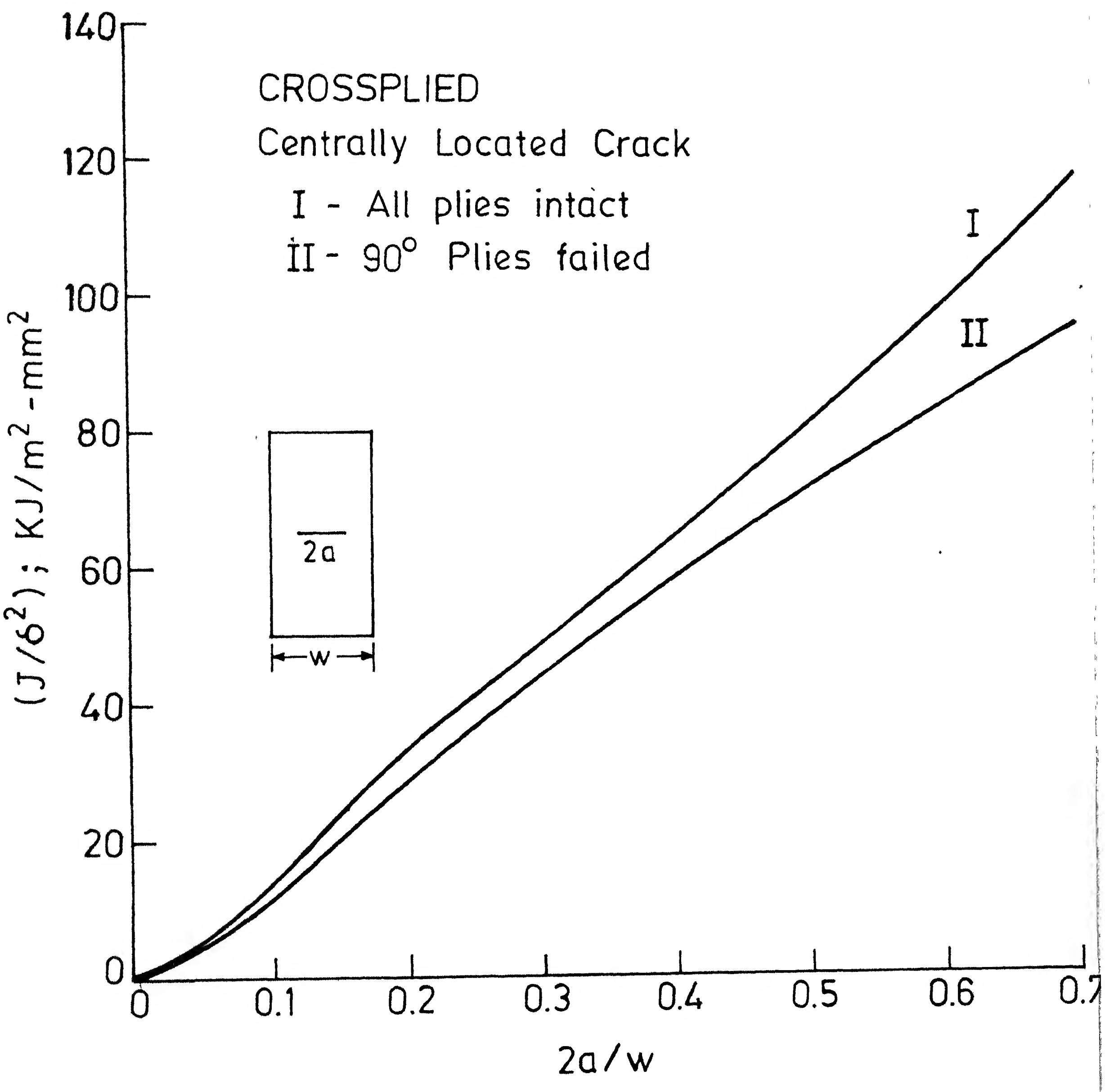

FIG. 3.12 J/δ^2 FOR CROSSPLIED LAMINATE.

The procedure adopted for evaluating J-integral of the laminate through the stepwise procedure can be explained as follows . Figure 3.13 shows an idealized load displacement curve of a specimen. The path OABC represents the model in which no stress relaxation takes place when a ply fails. The path OADEFG represents the model assuming complete load relaxation upon a ply failure. For the analysis here, a more practical case where a partial load relaxation takes place (represented by OAHIJK) has been assumed. The slopes c_1, c_2 and c_3 of the load-displacement curve segments between ply failures depend only on the initial crack length and are independent of the extent of load relaxation. If η_i denotes a load relaxation factor at a ply failure it can be defined as follows (with reference to Fig. 3.13).

$$\eta_1 = \frac{AH}{AD}$$

$$\eta_2 = \frac{IJ}{IF} \tag{3.4}$$

To obtain J-integral at a given displacement 'δ', area under the load displacement curve is first obtained . The three areas shown shaded in Fig. 3.13 can be easily obtained as

$$A_1 = \frac{1}{2} c_1 \delta_1^2$$

$$A_2 = \frac{1}{2} c_2 (\delta^2 - \delta_1^2)$$

$$A_3 = (1-\eta_1)(c_1-c_2)\,\delta_1(\delta-\delta_1) \tag{3.5}$$

So the expression for potential energy can be written as

$$U = A_1 + A_2 + A_3 = \frac{1}{2} c_1 \delta_1^2 + \frac{1}{2} (\delta^2 - \delta_1^2) + (1-\eta_1)(c_1-c_2)\delta_1 (\delta-\delta_1) \tag{3.6}$$

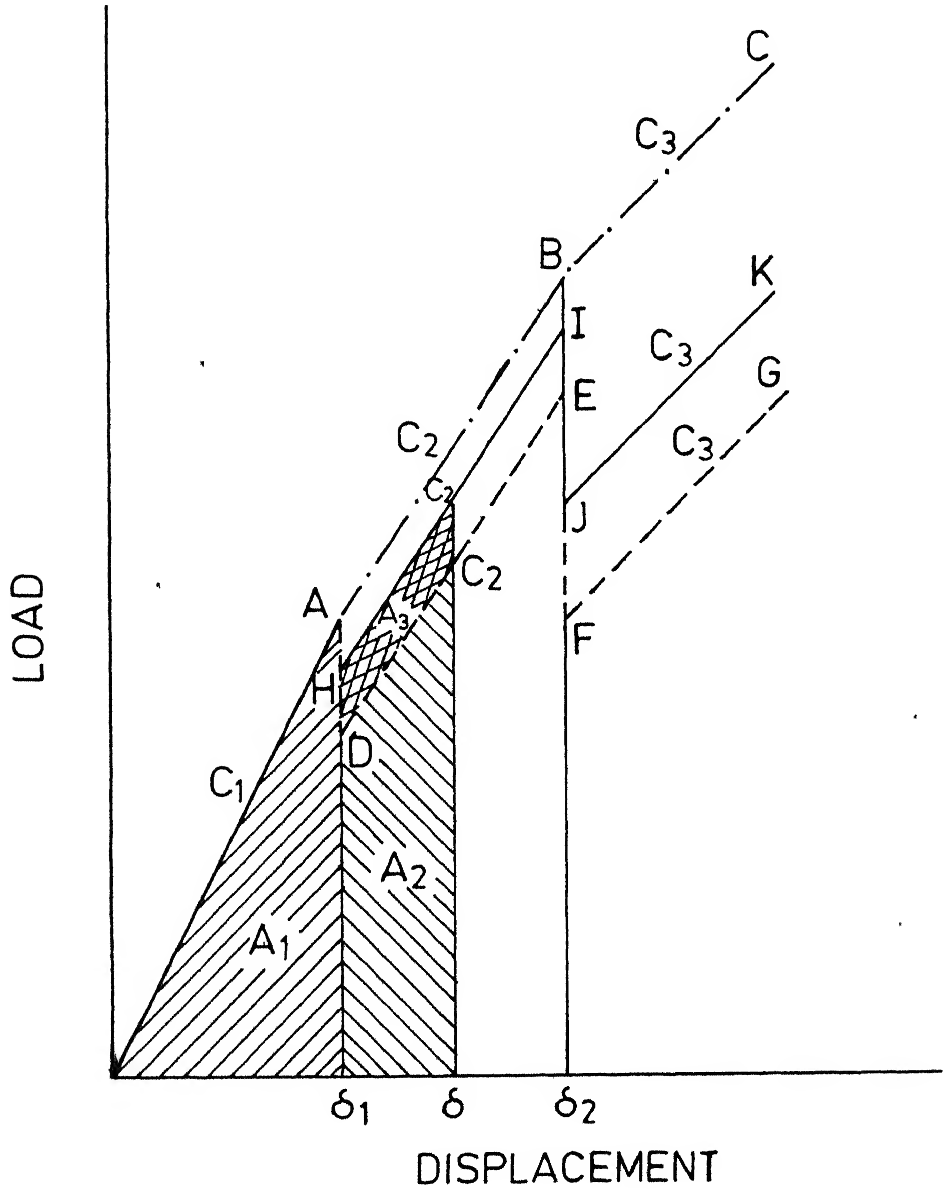

FIG. 3.13 LOAD-DISPLACEMENT CURVE FOR A LAMINA
WITH PLY FAILURE.

Now J-integral is given by $J = -\frac{\partial U}{\partial a}$, so that

$$J = -\frac{1}{2}\frac{\partial c_1}{\partial a}\delta_1^2 - \frac{1}{2}\frac{\partial c_2}{\partial a}(\delta^2-\delta_1^2) - (1-\eta_1)\left(\frac{\partial c_1}{\partial a} - \frac{\partial c_2}{\partial a}\right)\delta_1(\delta-\delta_1) \qquad (3.7)$$

Defining $k_1 = -\frac{1}{2}\frac{dc_1}{da}$ and $k_2 = -\frac{1}{2}\frac{dc_2}{da}$

where k_1 and k_2 are the values of (J/δ^2) in these segments,

$$J= k_1\delta_1^2 + k_2(\delta^2 - \delta_1^2) + 2(1-\eta_1)(k_1-k_2)\delta_1(\delta-\delta_1)$$

$$= k_2\delta^2 + (k_1-k_2)\delta_1^2 + 2(1-\eta_1)(k_1-k_2)\delta_1(\delta-\delta_1) \qquad (3.8)$$

So if 'δ' is in n^{th} segment then

$$J=k_n\delta^2+\sum_{i=2}^{n}\left[(k_{i-1}-k_i)\delta_{i-1}^2 + 2(1-\eta_{i-1})(k_{i-1}-k_i)\delta_{i-1}(\delta-\delta_{i-1})\right] \qquad (3.9)$$

Equation 3.9 has been used to obtain $J-\delta$ plot for the laminates.

3.3 COMPARISON WITH THE EXPERIMENTAL RESULT

Predictions of J_c for SEN specimens could be compared with the experimental results which are available only for SEN specimens [20]. Srinivasan [20] found that J vs δ curves for quasi-isotropic as well as crossplied laminates are independent of crack length when $\frac{a}{W} \geq 0.5$. The experimental value of J_c for a laminate is obtained from this common $J-\delta$ curve. In the present study, as has already been pointed out, $J-\delta$ curves for SEN specimens, though change with crack length, are quite close to each other for $a/w > 0.5$.

In the present study J_c values for the laminates has been obtained for three different models namely (i) when all the plies fail simultaneously (ii) when plies fail successively with no load relaxation in failed plies , and (iii) when plies fail successively with complete load relaxation in failed plies.

J vs curves for these three models in the range $0.5 \leq a/w \leq 0.7$ are shown in Figs. 3.14 to 3.19 for quasi-isotropic and crossplied laminates. Values of J_c obtained through different models are given in Tables 3.8 and 3.9 alongwith the experimental values. It is observed that the experimental and theoretical curves are quite comparable. However, the model in which all plies fail simultaneously predicts a higher value of J_c than the experimental value. This is expected because the off-axis plies do fail before the final fracture of the laminate. This model doesnot require a step-wise prediction method to be applied and hence is simple to apply.

The models in which plies fail successively predict lower values of J_c than the experimental. But the model which assumes no load relaxation in failed plies predicts J- δ curve and J_c value closes to the experimental ones. The difference between the predicted values of J_c through the models with no load relaxation and with complete load relaxation is about 10% and as may be expected former model predicts a higher value of J_c. Thus it may be concluded that it is basically the extensional stiffness matrix of the

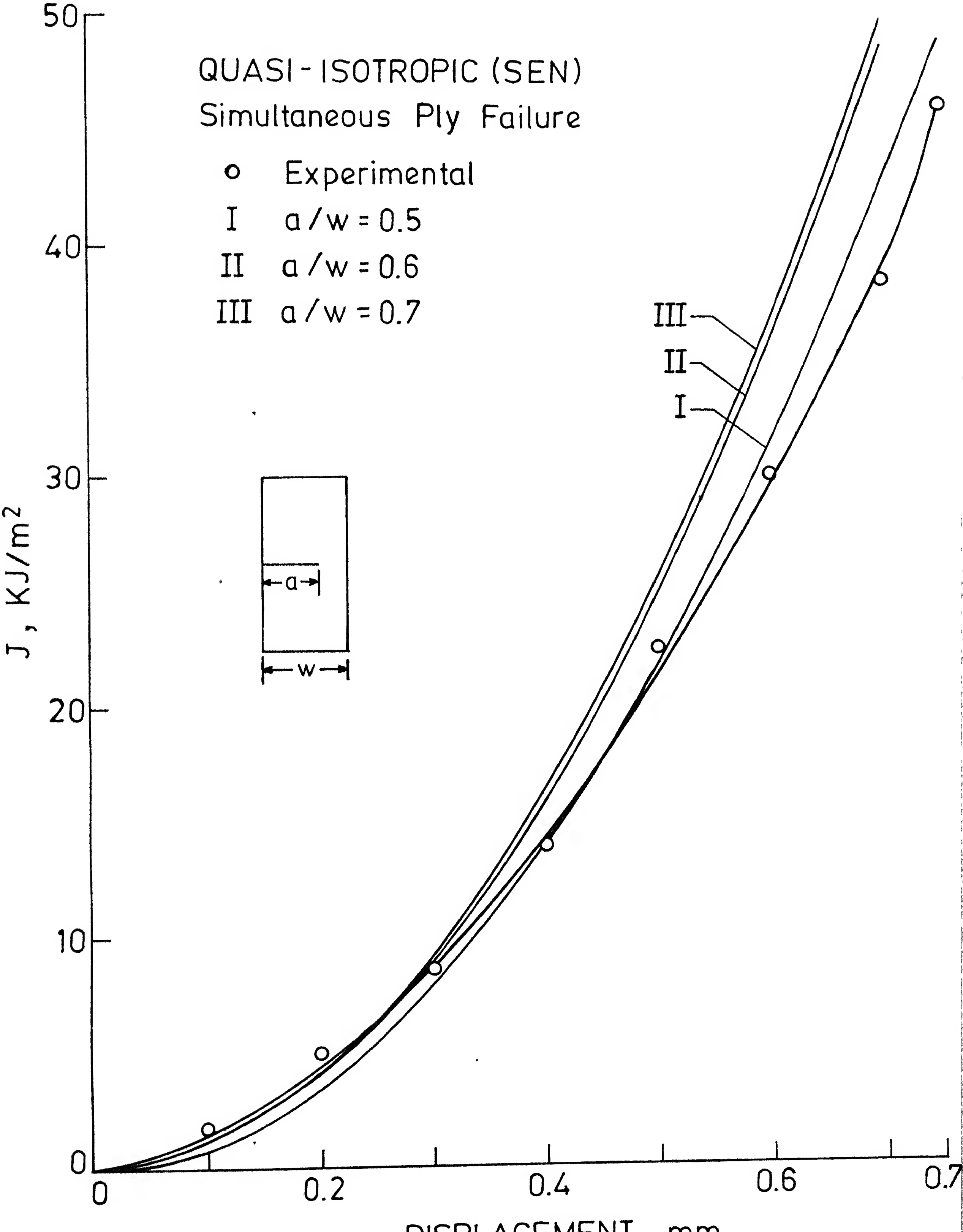

FIG. 3.14 J vs δ PLOT FOR QUASI-ISOTROPIC LAMINATE

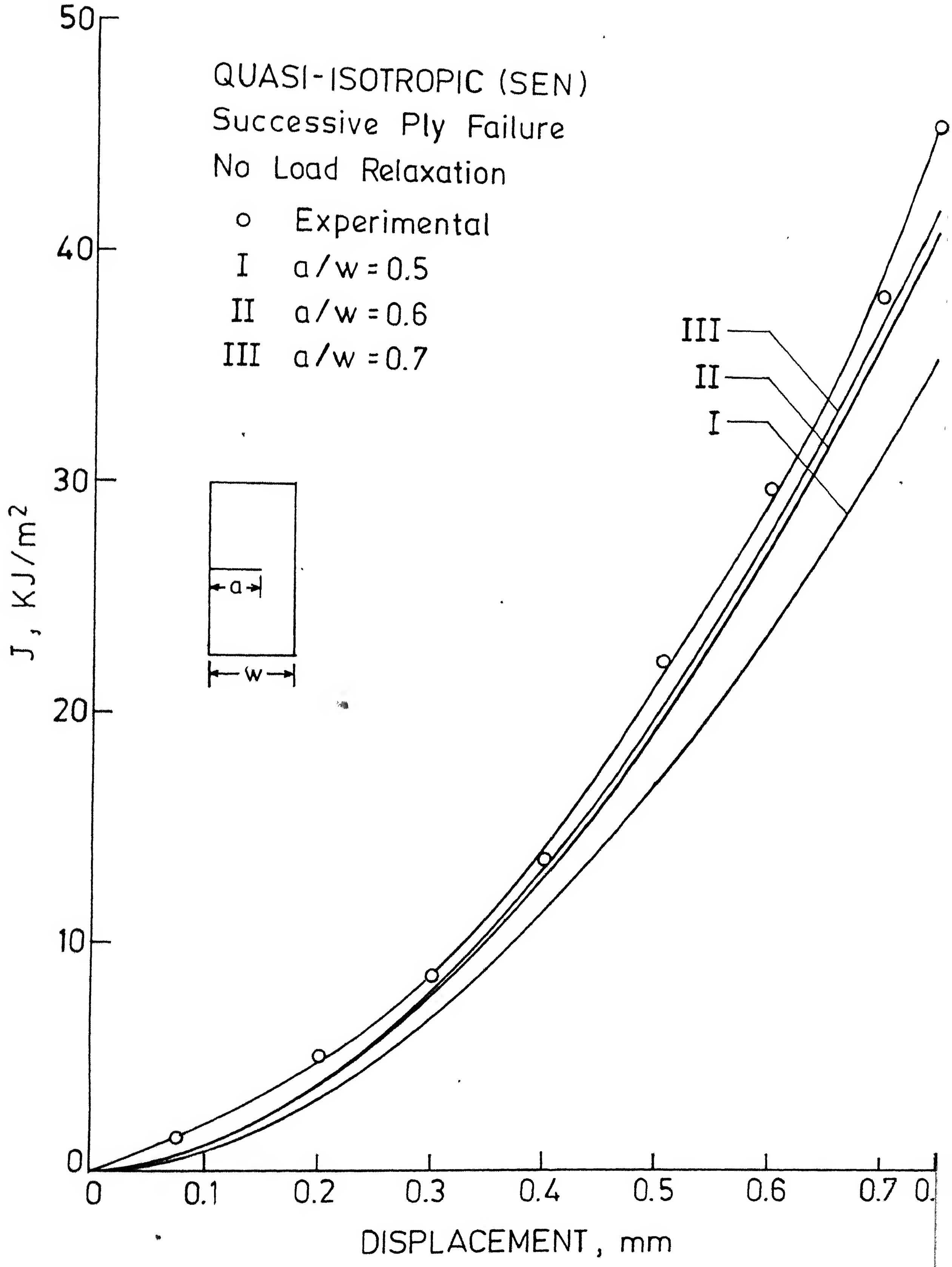

FIG. 3.15 J vs δ PLOT FOR QUASI-ISOTROPIC LAMINAT

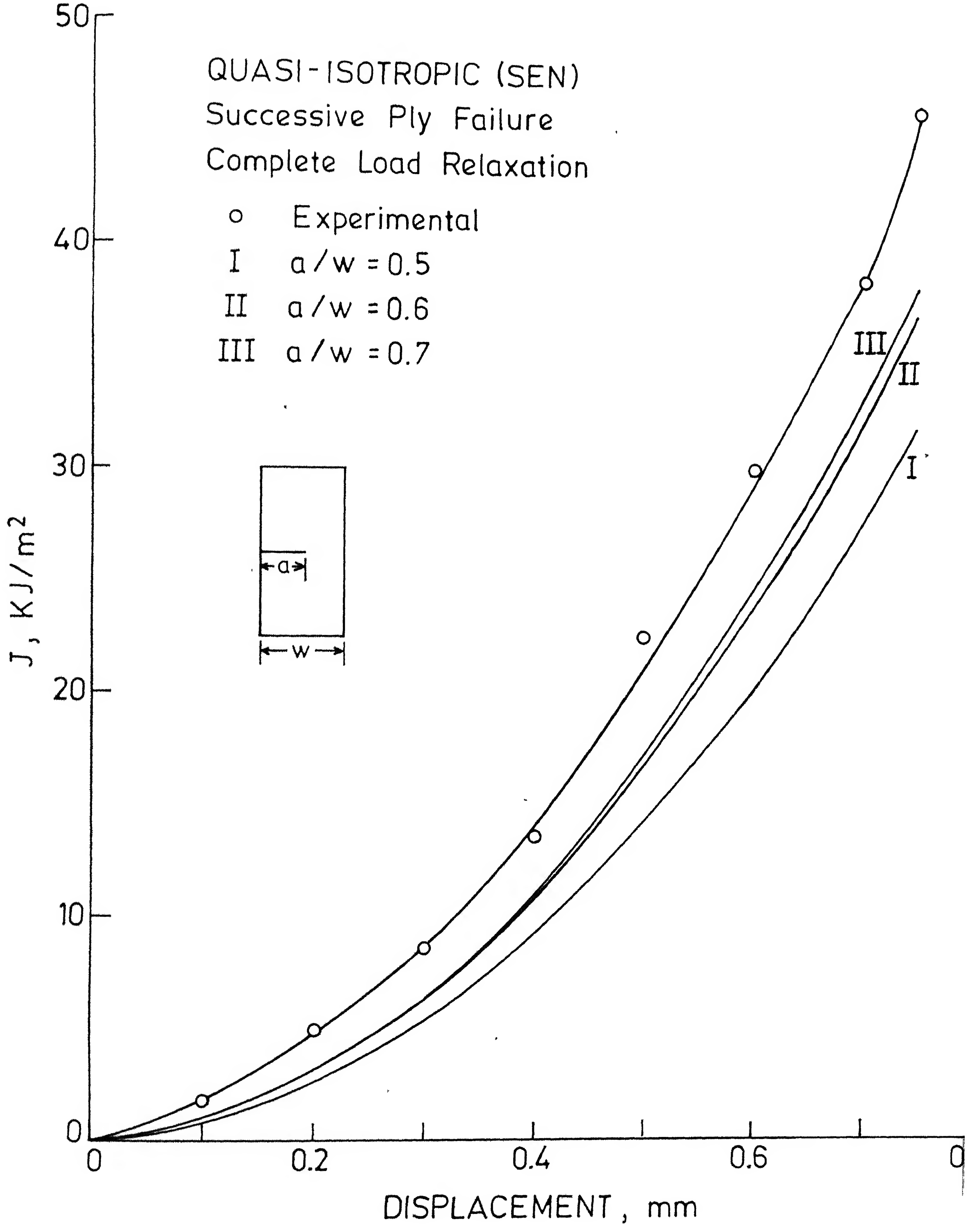

FIG. 3.16 J vs δ PLOTS FOR QUASI-ISOTROPIC LAMINATE

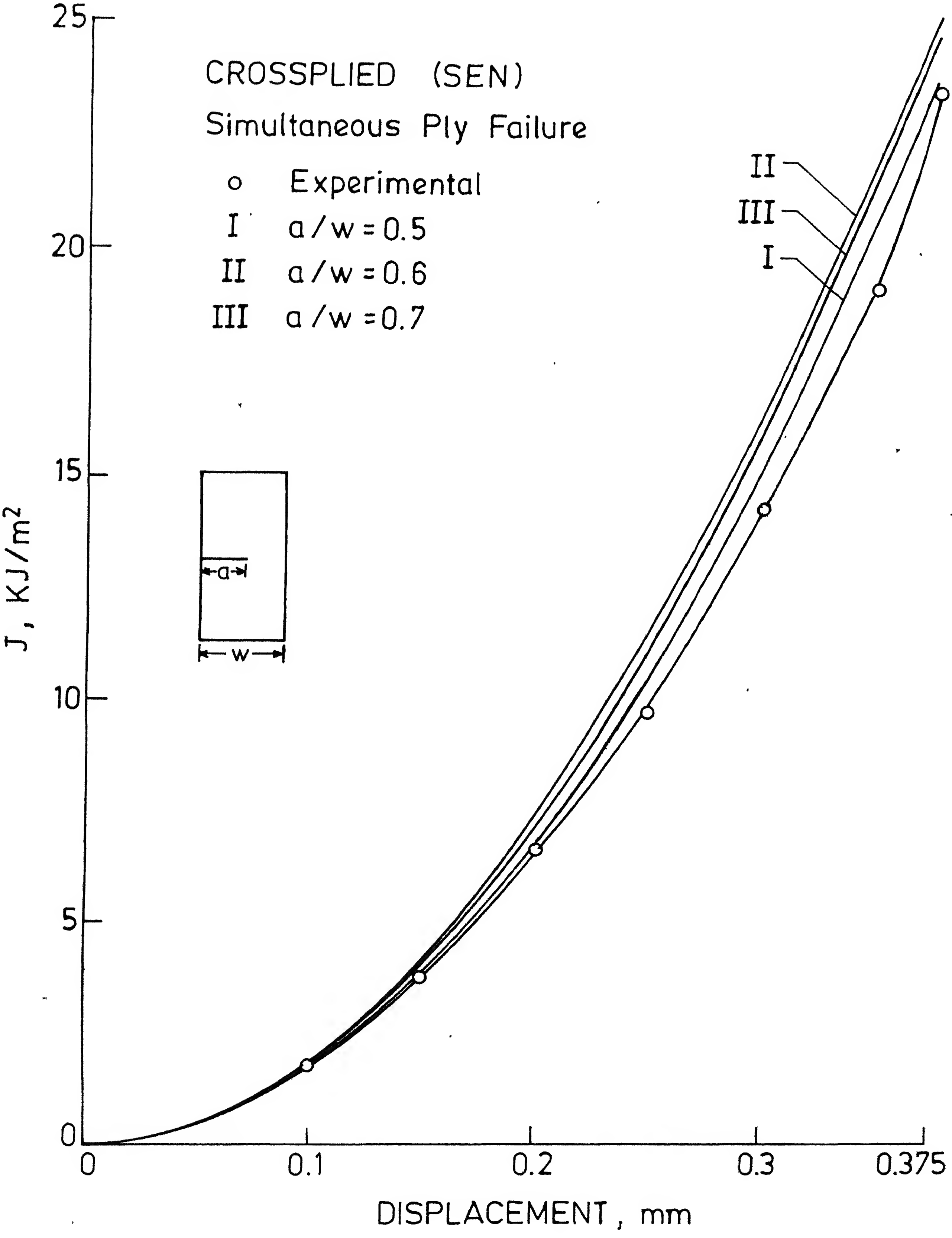

FIG. 3.17 J vs δ PLOT FOR CROSSPLIED LAMINATE.

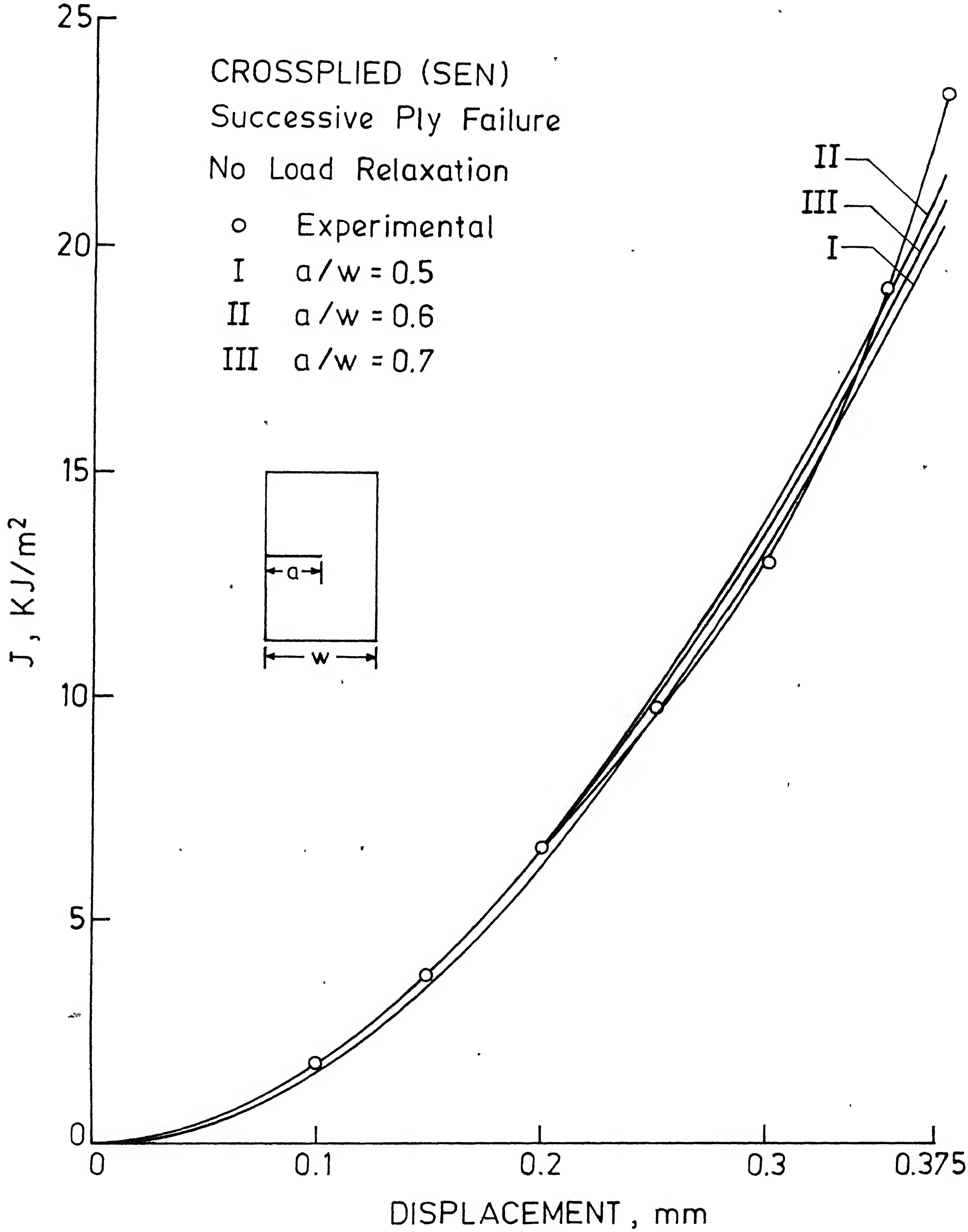

FIG. 3.18 J vs δ PLOT FOR CROSSPLIED LAMINATE.

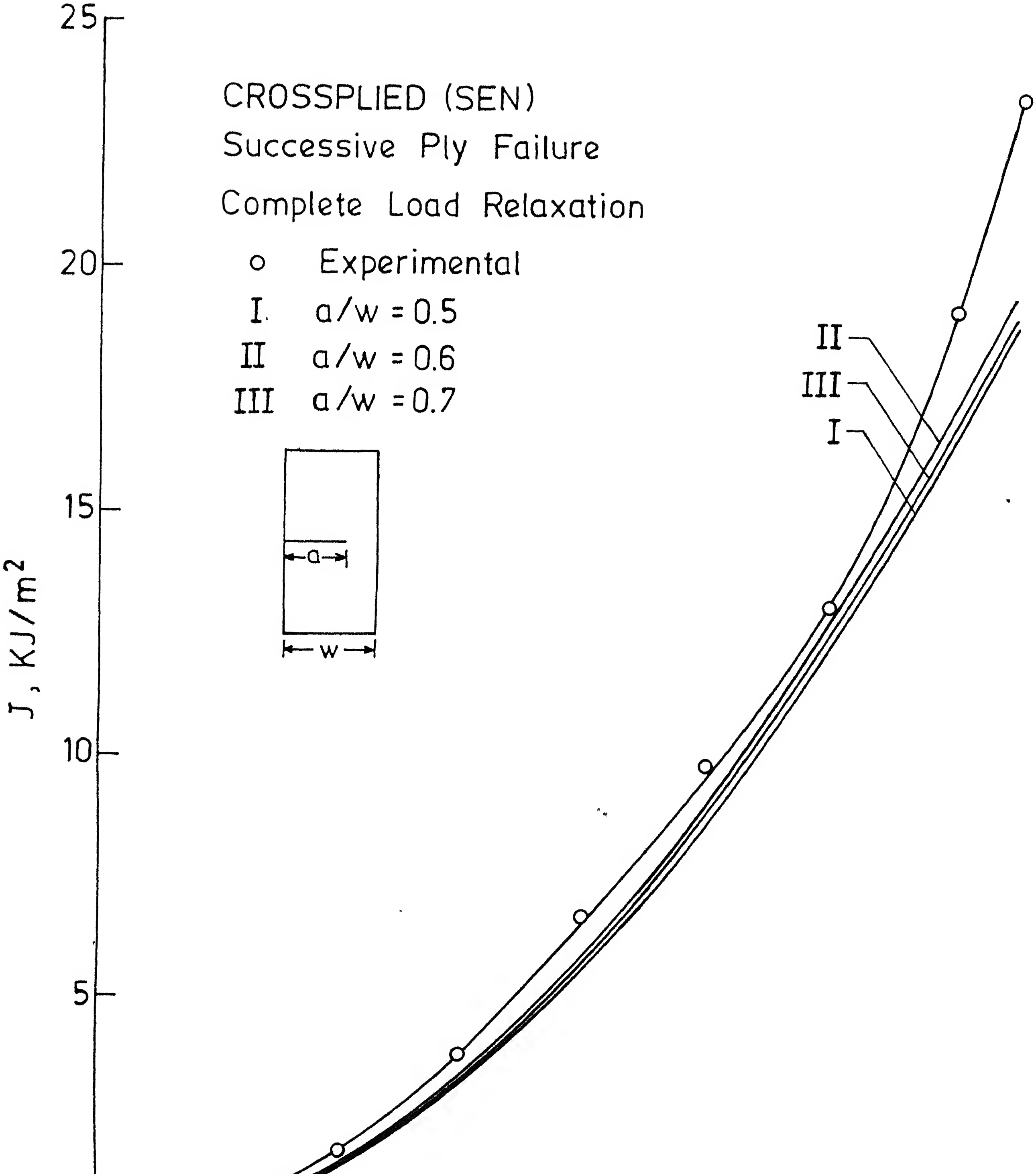

FIG. 3.19 J vs δ PLOTS FOR CROSSPLIED LAMINATE

TABLE-3.8

Quasi-isotropic Laminate

$\frac{a}{w}$	J_c in kJ/m^2			
	Present work			Experimental [20]
	Simultaneous ply failure	Successive ply failure		
		no load relaxation	Complete load relaxation	
0.5	49.25	35.75	31.95	
0.6	56.24	41.30	37.10	46.5
0.7	57.37	42.46	38.25	

TABLE -3.9

Crossplied Laminate

a/w	J_c in kJ/m^2			
	Present work			Experimental [20]
	Simultaneous ply failure	Successive ply failure		
		no load relaxation	complete load relaxation	
0.5	23.95	20.58	18.76	
0.6	25.49	21.56	19.44	22.65
0.7	25.11	21.16	19.03	

laminate that controls its fracture toughness and stacking sequence may not influence the fracture toughness significantly. The lower value of J_c predicted by step-wise procedure may be attributed to the fact that in these models, E_T and G_{LT} of the failed plies have been assumed to be zero and so in effect these layers carry no additional load in transverse direction. However, in a practical situation these layers do carry additional transverse load away from the dominant crack.

It may be pointed out that the final fracture of the laminate was assumed to occur at the experimentally obtained critical displacement of the laminate and not at the critical displacement of the 0° plies. Although this limits the applicability of the step-wise procedure, it is a necessary assumption in view of the experimental observations. Actually this is a consequence of a change in the critical displacement of 0° plies due to prior failure of off-axis plies and the resulting stress concentrations.

CHAPTER IV

CONCLUSIONS AND SUGGESTION FOR FUTURE WORK

J-integral for quasi-isotropic $(0/\pm 45/90)_{2s}$ and crossplied $(0/90)_{4s}$ laminates has been determined by finite element technique.

A stepwise analysis procedure was employed to obtain J_c. After a ply failure stiffness matrix of the failed ply was modified for further analysis. Models with no load relaxation and complete load relaxation in failed plies were investigated.

Based on the results presented in Chapter III the following conclusions are made.

(i) J- δ curve obtained for $0.5 \leq a/w \leq 0.7$ for SEN specimens are quite close to each other. This compares favourably with experimental finding of an unique J- δ curve for $a/w \geq 0.5$.

(ii) J_c values obtained by all the three models are in good agreement with experimental result. The model in which all plies fail simultaneously predicts a higher value of J_c than experimental value. Models in which plies fail successively predict lower values of J_c. The model in which no load relaxation in failed ply is assumed.,fits the experimental results better than the other two models.

(iii) Difference between J_c values for the two models with plies failing successively is about 10%. So that the effect of load relaxation in failed plies on J_c is marginal. This leads to the conclusion that extensional stiffness matrix of the laminate controls the fracture toughness and stacking sequence may not influence it significantly.

4.1 SUGGESTIONS FOR FUTURE WORK

(i) Experimental investigation for DEN and centrally located crack specimens should be conducted to find out J-integral for these specimens.

(ii) A three dimensional analysis of laminates may be carried out to understand the effect of interlaminar stresses and stacking sequence on J_c of the laminate.

(iii) A failure criterion to predict ply failure in a cracked specimen should be established.

(iv) J-integral should be obtained for laminates with off axis plies only, e.g., angle ply laminates ($\pm\theta$) and models presented in the present study should be compared.

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